**Latin Square Design (LSD)**

We have seen that in RBD, the whole experimental area is divided into homogeneous block and randomization is restricted once within the block. But in LSD, the whole experimental area is divided into certain rows and columns such that number of rows and columns are equal and each treatment occur only once in a row and column. In this design, treatments and shape is square due to equal number of treatments and replication, so called Latin Square Design. It is based upon the all the principles of design namely replication, randomization and Local Control.

**Layout:** Let us consider m treatments with m replication each so that there are N = mm = m2 experimental units. Let us consider the four treatments A, B, C, D and each with replication 4 times once in each row and column. The layout of LSD is.

|  |  |  |  |
| --- | --- | --- | --- |
| A | D | B | C |
| B | C | D | A |
| C | B | A | D |
| D | A | C | B |

**Mathematical model RBD:**

Let the linear model be

= μ + + + + ; i = 1, 2, ……, m; j = 1, 2, ……., m and k = 1, 2, ……….., m.

Where, = row and block of treatment.

μ = general mean effect (constant effect).

= effect due to row.

= effect due to block or column.

= effect due to treatment.

= error due to chance.

**Assumptions of RBD:**

1. All the observations are independent.
2. All the observations should be drawn from normal population having constant variance.
3. All the treatment should be homogeneous as for as possible.
4. Various treatments and environmental effects are additive in nature.
5. e i j k are independent of N(0, σe2 ).

**Problem:** To test

H 0R: μ1 = μ2 = ………. = μ m; there is no significance difference between rows.

H 1R: At least one row is different.

And,

H 0C: μ1 = μ2 = ………. = μ m; there is no significance difference between columns.

H 1C: At least one column is different.

And,

H 0T: μ1 = μ2 = ………. = μ m; there is no significance difference between treatments.

H 1T: At least one treatment is different.

**Statistical Analysis:**

Let the linear model be

= μ + + j + + ; i = 1, 2, ……, m; j = 1, 2, ……., m and k = 1, 2, …….., m ……..(1)

The parameters μ, j and are determined by using the principle of least square by minimizing error (residual) sum of square.

Total sum of square = Sum of square due to row + Sum of square due to column + Sum of square due to treatment + Sum of square due to error.

TSS = SSR + SSC + SST + SSE

**Degree of freedom for various sum of square of LSD:**

Degree of freedom of TSS = m2 – 1 = N - 1

Degree of freedom of SST = m – 1

Degree of freedom of SSR = m – 1

Degree of freedom of SSB = m – 1

Degree of freedom of SSE = (m-1) (m-2)

**Mean sum of squares:**

Mean sum of square due to row (MSR) =

Mean sum of square due to column (MSC) =

Mean sum of square due to treatment (MST) =

Mean sum of square due to error (MSE) =

**ANOVA Table of LSD:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of variation | d f | SS | MS | F-Ratio |
| 1. Due to row 2. Due to column 3. Due to treatment 4. Due to error | m – 1  m - 1  m - 1  (m–1)(m – 2) | SSR  SSC  SST  SSE | MSR =  MSC =  MST =  MSE = | FR =  FC =  FT = |
| Total | m2 - 1 | TSS |  |  |

**Critical region:**

The tabulated value of F at α% for treatment is F α, [m – 1, (m - 1)(m – 2)].

**Decision:**

If F Cal. FTab; we accept H0. Otherwise reject H0.

**Computational formula:**

C.F. =

TSS = - C.F.

SSR =

SSC =

SST =

SSE = TSS – SSR – SSC – SST

**Advantage of LSD:**

1. With the use of two way classification, LSD controls more variation than CRD or RBD.
2. The analysis of the design is simple and straight forward.
3. The analysis remains simple even if some observations are missing.
4. LSD is an incomplete three way layout. It’s advantage over the complete three way layout is that instead of m3experimental units only m2 units are needed.

**Disadvantage of LSD:**

1. The process of randomization is very tedious.
2. In this design the number of replication is necessarily equal to number of treatments.
3. It is suitable for treatments 5 to 10.
4. It is not easy in the field layout.

**Uses of LSD:**

1. It is used in industrial concerns laboratories.
2. It is used in agricultural field.
3. It is used to control the two factors of variations simultaneously.

**Example:** The layout and yield of four treatments in a 4 4 experiment in shown in the following table. Analyze the data.

|  |  |  |  |
| --- | --- | --- | --- |
| D20 | B17 | A20 | C19 |
| B21 | A18 | C18 | D17 |
| A18 | C21 | D17 | B17 |
| C20 | D19 | B17 | A18 |

**Solution:** Here,

**Problem:** To test

H 0R: μ1 = μ2 = ………. = μ m; there is no significance difference between rows.

H 1R: At least one row is different.

And,

H 0B: μ1 = μ2 = ………. = μ m; there is no significance difference between columns.

H 1B: At least one column is different.

And,

H 0T: μ1 = μ2 = ………. = μ m; there is no significance difference between treatments.

H 1T: At least one treatment is different.

Calculation table is

|  |
| --- |
| Row total () |
|  | D20 | B17 | A20 | C19 | 76 |
| B21 | A18 | C18 | D17 | 74 |
| A18 | C21 | D17 | B17 | 73 |
| C20 | D19 | B17 | A18 | 74 |
| Column total () | 79 | 75 | 72 | 71 | G = 297 |

Also, Calculation table is

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Treatment |  |  |  |  | Total () |
| A | 20 | 18 | 18 | 18 | 74 |
| B | 17 | 21 | 17 | 17 | 72 |
| C | 19 | 18 | 21 | 20 | 78 |
| D | 20 | 17 | 17 | 19 | 73 |

m = 4, N = m2 = 42 = 16

C.F. = = = 5513.0625

TSS = - C.F.

= (20)2 + (18)2 + (18)2 + (18)2 + (17)2 + (21)2 + (17)2 + (17)2 + (19)2 + (18)2 + (21)2 + (20)2

+ (20)2 + (17)2 + (17)2 + (19)2 – 5513.0625

= 31.9375

SSR = = [(76)2 + (74)2 + (73)2 + (74)2] – 5513.0625 = 1.1875

SSC = = [(79)2 + (75)2 + (72)2 + (71)2] – 5513.0625 = 9.6875

SST = = [(74)2 + (72)2 + (78)2 + (73)2] – 5513.0625 = 5.1875

SSE = TSS – SSR – SSC – SST = 31.9375 – 1.1875 – 9.6875 – 5.1875 = 15.875

**ANOVA Table of LSD:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source of variation | d f | SS | MSS | F-Ratio | F Tab. |
| 1. Due to row 2. Due to column 3. Due to treatment 4. Due to error | 3  3  3  6 | 1.1875  9.6875  5.1875  15.875 | 0.3958  3.2292  1.7292  2.6458 | FR = 0.1496  FC = 1.2205  FT = 0.6536 | F 0.05, (3, 6) = 4.76  F 0.05, (3, 6) = 4.76  F 0.05, (3, 6) = 4.76 |
| Total | 15 | 31.9375 |  |  |  |

**Decision:** Since, F R < F 0.05, (3, 5) = 5.41; we accept H0, there is no significance difference between rows.

Since, F C < F 0.05, (3, 6) = 5.41; we accept H0, there is no significance difference between columns.

Since, F T < F 0.05, (3, 6) = 5.41; we accept H0, there is no significance difference between treatments.

**Question:** The following data represents the yields from manorial experiment with sugarcane. Carry out ANOVA of the following design.

|  |  |  |  |
| --- | --- | --- | --- |
| A 12 | C 19 | B 10 | D 8 |
| C 18 | B 12 | D 6 | A 7 |
| B 22 | D 10 | A 5 | C 21 |
| D 12 | A 7 | C 27 | B 17 |

**Question:** Set up the analysis of variance for the following results of a design.

|  |  |  |
| --- | --- | --- |
| A 10 | B 15 | C 20 |
| B 25 | C 10 | A 15 |
| C 25 | A 20 | B 15 |

**Missing value in LSD:**

Let us consider a m m LSD. Let one of the observation occurring in it hrow, j t h column and k t h treatment is missing. Let the missing observation be x. That is

x =

Where, m = No. of row = No. of column = No. of treatment.

= Sum of remaining value of row within the missing observations.

= Sum of remaining value of column within the missing observations.

= Sum of remaining value of treatment within the missing observations.

= Grand total of remaining value of all the observation within the missing observations.

Substitute the value of x in place of missing value and carry out analysis as usual except that one degree of freedom is subtracted from total and consequently from error. Because of change in level of degree of freedom, we obtain an upward bias in SST. Hence to get better result subtracted an adjustment factor from SST.

Adjustment factor (k) =.

Adjusted SST (SSTA) = SST – k.

**Example:** Determine the missing value and carry out ANOVA of the following design.

|  |  |  |  |
| --- | --- | --- | --- |
| D20.1 | B19.4 | C30.6 | A7.9 |
| C17.5 | A10.4 | D21.2 | B19.1 |
| A? | D18.1 | B24.6 | C25.2 |
| B25.1 | C30.4 | A10.2 | D28.0 |

**Solution:** Here,

Let the missing value be x.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Calculation table is | | | |  |
| D 20.1 | B 19.4 | C 30.6 | A 7.90 | 78 |
| C 17.5 | A 10.4 | D 21.2 | B 19.1 | 68.2 |
| A x | D 18.1 | B 24.6 | C 25.2 | 67.9 + x |
| B 25.1 | C 30.4 | A 10.2 | D 28.0 | 93.7 |
|  | 62.7 + x | 78.3 | 86.6 | 80.2 | G = 307.8 + x |

m = 4, = 67.9, = 62.7, = 10.4 + 10.2 + 7.90 = 28.5 and = 307.8

Therefore,

Missing value (x) =

=

= 3.46

Now,

**Problem:** To test

H 0R: μ1 = μ2 = ………. = μ m; there is no significance difference between rows.

H 1R: At least one row is different.

And,

H 0B: μ1 = μ2 = ………. = μ m; there is no significance difference between columns.

H 1B: At least one column is different.

And,

H 0T: μ1 = μ2 = ………. = μ m; there is no significance difference between treatments.

H 1T: At least one treatment is different.

Then,

G = 307.8 +x+ = 307.8 + 3.46 = 311.26

N = m2 = 42 =16

C.F. = = = 6055.17

= (20.1)2 + (19.4)2 + (30.6)2 + (7.9)2 + (17.5)2 + (10.4)2 + (21.2)2 + (19.1)2 + (3.46)2 +

(18.1)2 + (24.6)2 + (25.2)2 + (25.1)2 + (30.4)2 + (10.2)2 + (28.0)2

= 7029.79

TSS = - C.F. = 7029.79 – 6055.17 = 974.62

SSR = = [(78)2 + (68.2)2 + (67.9+x) 2 + (93.7)2 – 6055.17 = 96.62

SSC = = [(62.7 + x) 2 + (78.3)2 + (86.6) 2 + (80.2)2 – 6055.17 = 54.73

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Treatment |  | | | | T..k |
| A | 3.46 | 10.4 | 10.2 | 7.9 | 31.96 |
| B | 25.1 | 19.4 | 24.6 | 19.1 | 88.2 |
| C | 17.5 | 30.4 | 30.6 | 25.2 | 103.7 |
| D | 20.1 | 18.1 | 21.2 | 28.0 | 87.4 |

SST = = [(31.96)2 + (88.2)2 + (103.7) 2 + (87.4)2 – 6055.17 = 743.11

Adjustment factor (k) = = = 233.58

Adjusted SST (SSTA) = SST – k = 743.11 – 233.58 = 509.53

SSE = TSS – SSR – SSC – SSTA = 974.62 – 96.62 – 54.73 – 743.11 = 0.68

**ANOVA Table of LSD:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source of variation | d f | SS | MSS | F-Ratio | F Tab. |
| 1. Due to row 2. Due to column 3. Due to treatment 4. Due to error | 3  3  3  5 | 96.62  54.73  509.53  0.68 | 32.2  18.24  169.84  0.136 | FR = 236.76  FC = 134.11  FT = 1248.82 | F 0.05, (3, 5) = 5.41  F 0.05, (3, 5) = 5.41  F 0.05, (3, 5) = 5.41 |
| Total | 15 | 974.62 |  |  |  |

**Decision:** Since, F R > F 0.05, (3, 5) = 5.41; there is significance difference between rows.

Since, F C > F 0.05, (3, 5) = 5.41; there is significance difference between columns.

Since, F T > F 0.05, (3, 5) = 5.41; there is significance difference between treatments.

**Question:** The table given below represents the yields of 4 varieties in a 4 replicate experiment for which one observation is missing. Estimate the missing value and then carry out the ANOVA.

|  |  |  |  |
| --- | --- | --- | --- |
| A 12 | C 19 | B 10 | D 8 |
| C 18 | B 12 | D 6 | A ? |
| B 22 | D 10 | A 5 | C 21 |
| D 12 | A 7 | C 27 | B 17 |

**Efficiency of LSD relative to CRD:**

The precision of LSD is compared to precision of CRD is called efficiency of LSD relative to CRD, then

Efficiency (E) = = = = =

Interpretation:

If then LSD less efficient than CRD

If then LSD more efficient than CRD

If then LSD and CRD are equally effective

Example: From the following ANOVA table of 4 4 LSD determine it’s efficiency with respect to CRD.

|  |  |  |  |
| --- | --- | --- | --- |
| Source of variation | d f | SS | MSS |
| 1. Due to row 2. Due to column 3. Due to treatment 4. Due to error | 3  3  3  6 | 1.1875  9.6875  5.1875  15.875 | 0.3958  3.2292  1.7292  2.6458 |
| Total | 16 | 31.9375 |  |

Solution: Given,

= 4, MSR = 0.3958, MSC = 3.2292, MSE = 2.6458

Efficiency (E) =?

Now,

Efficiency (E) = = = 2.674

Hence, LSD is (2.674 – 1) = 1.674 = 1.674 100% = 167.4% more efficient than CRD.

**Efficiency of LSD relative to RBD:**

The precision of LSD is compared to precision of RBD is called efficiency of LSD relative to RBD, then

(i) When row is taken as block.

Efficiency (E) = = = = =

(i) When column is taken as block.

Efficiency (E) = = = = =

Interpretation:

If then LSD less efficient than RBD

If then LSD more efficient than RBD

If then LSD and RBD are equally effective

Example: From the following ANOVA table of 4 4 LSD determine it’s efficiency with respect to RBD.

|  |  |  |  |
| --- | --- | --- | --- |
| Source of variation | d f | SS | MSS |
| 1. Due to row 2. Due to column 3. Due to treatment 4. Due to error | 3  3  3  6 | 2.133  2.203  10.663  7.059 | 0.711  0.734  3.554  1.177 |
| Total | 15 | 22.058 |  |

Solution: Given,

= 4, MSR = 0.711, MSC = 0.734, MSE = 1.177

Efficiency (E) =?

Now,

(i) When row is taken as block.

Efficiency (E) = = = 0.905 = 90.5%

Hence, LSD is (100 – 90.5) % = 9.5% less efficient than RBD when row taken as block.

(ii) When column is taken as block.

Efficiency (E) = = = 0.901 = 90.1%

Hence, LSD is (100 – 90.1) % = 9.9% less efficient than RBD when column taken as block.

**Numerical problems:**

15. The table given below is yields of 3 varieties in a 4 replicate experiment for which one observation is missing. Estimate the missing value and carry out the ANOVA.

|  |  |  |  |
| --- | --- | --- | --- |
| A 12 | C 19 | B 10 | D 8 |
| C 18 | B 12 | D 6 | A ? |
| B 22 | D 10 | A 5 | C 21 |
| D 12 | A 7 | C 27 | B 17 |

[Ans: 2, = 1.61, = 0.85, = 8.96, Accept and]

16. Complete the following table for the analysis of variance of a design.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| SV |  | SS | MSS | F |
| Blocks | 4 | 26.8 | ? | ? |
| Treatment | 3 | ? | ? | ? |
| Error | ? | ? | 2.5 |  |
| Total | ? | 85.3 |  |  |

[Ans: 12, 19, 28.5, 30, 6.7, 9.5, 2.68, 3.8]

17. Fill in the blanks in the following analysis of variance table of a design.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| SV |  | SS | MSS | F |
| Rows | ? | 72 | ? | 2 |
| Columns | ? | ? | 36 | ? |
| Treatment | ? | 180 | ? | ? |
| Error | 6 | ? | 12 |  |
| Total | ? | ? |  |  |

[Ans: 3, 3, 3, 15, 108, 72, 432, 24, 60, 3, 5]

18. Complete the following table for analysis of variance a design.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| SV |  | SS | MSS | F |
| Columns | 5 | ? | ? | ? |
| Rows | ? | 4.2 | ? | ? |
| Treatment | ? | ? | 2.43 | ? |
| Error | ? | ? | 0.65 |  |
| Total | ? | 39.65 |  |  |

The columns as representing schools, the rows as classes, the treatments as methods of teaching and the observations as grades based on 100 points. Test the hypothesis that the treatment effects are equal to zero.

[Ans: 5, 5, 20, 35, 10.3, 12.15, 13, 2.06, 0.84, 3.16, 1.29, 3.73, Reject]

19. Consider the partially completed ANOVA table below. Complete the ANOVA table and answer the followings. What design was employed? How many treatments were compared? How many observations were analyzed? At 0.05 level of significance can one conclude that the treatments have different effects? Why?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| SV |  | SS | MSS | F |
| Treatment | 2 | 231.5 | ? | ? |
| Blocks | 7 | ? | ? | ? |
| Error | ? | 573.75 | ? |  |
| Total | 23 | 903.75 |  |  |

[Ans: 98.5, 14, 115.75, 14.07, 40.98, 2.82, 0.34, Acept]

20. From the following ANOVA table of RBD, determine it’s efficiency with respect to CRD.

|  |  |  |  |
| --- | --- | --- | --- |
| SV |  | SS | MSS |
| Between blocks | 15 | 21.55 | 4.31 |
| Between treatment | 3 | 15.66 | 5.22 |
| Error | 15 | 12.3 | 0.82 |
| Total | 23 | 49.51 |  |

[Ans: 1.925]

21. From the following ANOVA table of LSD, determine it’s efficiency (i) with respect to CRD, (ii) with respect to RBD when columns are taken as blocks, and, (iii) with respect to RBD when rows are taken as blocks.

|  |  |  |  |
| --- | --- | --- | --- |
| SV |  | SS | MSS |
| Rows | 3 | 259.5375 | 86.4375 |
| Columns | 3 | 155.2725 | 51.7575 |
| Treatments | 3 | 1372.1225 | 457.3742 |
| Error | 6 | 156.3700 | 26.0616 |
| Total | 15 | 1943.0775 |  |

[Ans: 1.6605, 1.2464, 1.5791]

**Thank you!!!**